A Two-Parameter Model for a Nonideal Flow Reactor

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There are two well-known methods of handling the phenomenon of nonideal flow. One is the "fluid particle method," first presented by MacMullin and Weber (*1*), and the other is the "method of models" (*1* to *4*). Some of the more common models presented in the literature are the equivalent series of tanks (*1*), the plug flow with recycle (*2*), and the dispersion model (*3*).

A comprehensive treatment of the plug flow with recycle model is presented by Gillespie and Carberry (*2*). This model is not the one treated in this paper.

A two parameter model is presented in this manuscript which would account for residence time distribution data which falls between the discrete n values of the series of stirred tanks model. This model is the series of stirred tanks with recycle. The general differential equation for the response to a step function is derived.

THE SERIES OF TANKS WITH RECYCLE MODEL

The purpose of this paper is to define a recycle model and to study its properties and uses. Let us first consider any system which can be represented in terms of a series of continuous stirred tank reactors. The residence time distribution curves (f-curves) for a system of n stirred tanks are discrete curves. Thus, if an experimental f-curve lies between the curves for $n = i$ and $n = i + 1$, we must choose the closer line. The recycle model gives us a continuous family of solutions so that an f-curve may be matched very accurately.

To formulate this model, consider a series of n continuous stirred tanks of equal volume with a recycle from the product line which mixes with the feed stream.

Mole balances may be written for each tank.

$$\theta_s = \frac{nV}{F} \tag{1}$$

$$C_{1-1} = \frac{C_F + RC_n}{1+R} \tag{2}$$

$$C_{i-1} = C_i + \frac{\theta_s}{n(1+R)} C_i' \tag{3}$$

for $i = 1, 2, 3, \ldots n$.

(Note: C_{1-1} must be distinguished from C_o, which will be used later).

This system of differential equations may be solved by repeatedly differentiating and substituting C_{i-1} into the equation for C_{i-2}.

This leads to the equation

$$u(t) = f(T) + \sum_{i=1}^{n} \binom{n}{i} \frac{f^{(i)}(T)}{n^i (1+R)^{i-1}} \tag{4}$$

Equation (4) may be solved by applying the initial conditions which state that the function is equal to unity and each derivative is equal to zero for all tanks.

If $R = 0$, the system is simply n stirred tanks. If R approaches infinity, Equation (4) becomes

$$u(t) = f + f' \tag{5}$$

which is the differential equation for one stirred tank. If R has any real value greater than zero, the f-curve defined will lie between the curves for n stirred tanks and one stirred tank.

This model is important because a continuous spectrum of f-curves can be produced between n stirred tanks and one tank. This model is most useful for low values of n. All the f-curves obtained from this model are readily interpretable in terms of a simple system and two simple constants. It is also worthy of note that for n greater than 2, R need not go all the way to infinity as there should be a value for which the f-curve will lie on the $n - 1$ curve.

As an example of the use of the recycle model, we may determine by some means that an experimental f-curve lies between the curves for $n = 2$ and $n = 1$. We then pick the larger n and evaluate Equation (4).

$$u(t) = f(T) + f'(T) + \frac{1}{4(1+R)} f''(T) \tag{6}$$

The initial conditions are

$$f(0) = u(t)$$

$$f'(0) = 0$$

This is a second order, homogeneous differential equation. The form of the solution is

$$f = c_1 e^{-aT} + c_2 e^{-bT}$$

where

$$-a = -2(1+R) - 2\sqrt{R+R^2}$$

$$-b = -2(1+R) + 2\sqrt{R+R^2}$$

Applying initial conditions, the solution is

$$f = \left(\frac{1}{a-b}\right)(ae^{-bT} - be^{-aT})$$

Or, substituting,

$$f = \left[\frac{1}{2\sqrt{R+R^2}}\right][\sqrt{R+R^2} + (1+R)]\, e^{-2[(1+R) - \sqrt{R+R^2}]T} + [\sqrt{R+R^2} - (1+R)]\, e^{-2[(1+R) + \sqrt{R+R^2}]T} \tag{7}$$

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We have a continuous family of curves between the curve for two tanks and that for one tank. The curves for low R approach zero more quickly, as predicted. The curves dip downward from zero slope sooner as R increases, and the inflection points move up the curve.

APPLICATION OF THE MODEL

The model is most useful for cases where the vessel being studied would correspond to a series of tanks with a low value of n but where data lie between the curves for two discrete values of n. If the corresponding number of tanks in the series is high (let us say, about ten), it would be sufficiently accurate to take the next higher or lower n. If the value falls between one and two, two and three, three and four, and so forth, one would choose the higher value of n and then select a value of R to fit the experimental data.

SUMMARY

A two parameter model for a nonideal flow reactor was presented. The equations for the residence time distribution curves were derived. The advantages of this model were described.

NOTATION

C = concentration
F = volumetric flow rate
R = recycle ratio
t = time
T = reduced time, t/θ
V = volume of one tank in the series
f = transient response function
i = any given tank in series
n = number of tanks in series
$u(t)$ = unit step function
θ_s = normal holding time of the system (V/F)

LITERATURE CITED

1. MacMullin, R. B., and H. Weber, *Trans. AIChE*, **31**, 409 (1935).
2. Gillespie, B., and J. J. Carberry, *Ind. Eng. Chem., Fundamentals*, **5**, No. 2, 164 (May, 1966).
3. Levenspiel, Octave, "Chemical Reaction Engineering," Chapter 10, Wiley, New York (July, 1962).
4. Denbigh, Kenneth, "Chemical Reactor Theory," p. 152, Cambridge University Press, London (1965).

Magnetohydrodynamics of Liquid Films Flowing along a Vertical Plane Surface

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The problem of why when a vessel of liquid has been emptied and set aside, a thin film of liquid clings to the inside and gradually drains down to the bottom under the action of gravity, was posed and solved by Jeffreys (*1*). The problem, which has wide practical applications, has been reexamined very recently by Gutfinger and Tallmadge (*2*), who included the unsteady term, and by Hassan and Franzini (*3*), who included the inertia terms without the unsteady term in the equation of motion. In this problem of drainage we want to know the flow rate and the thickness of the liquid film draining down a surface. These variables depend upon the shape of the surface; the surface tension; and the gravitational, inertial, viscous, and other external forces if any. A study of the effect of an external force like $\vec{J} \times \vec{B}$ arising due to the interaction of electromagnetic forces and hydrodynamic forces on the viscous lifting of a conducting fluid film has been made by Bradshaw et al. (*4*).

We will consider here the effect of the hydromagnetic force $\vec{J} \times \vec{B}$ on the thickness of the draining liquid film of conductivity σ along a vertical plane surface when the unsteady term is also taken account of in the equation of motion. The inclusion of the unsteady term can be justified because steady conditions are impossible to obtain when film thickness changes continuously with time. On the assumption that the force $\vec{J} \times \vec{B}$ simplifies to $-\sigma B_o^2 u$, it is seen that the equations of the problem are identical to those for one-dimensional unsteady heat conduction with heat generation linear with temperature, as pointed out by the reviewer.

With the usual assumption the equation of fluid flow past a vertical plane surface in the presence of a magnetic field B_o imposed in the y direction is

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g - \frac{\sigma B_o^2 u}{\rho} \tag{1}$$

and the equation of continuity is

$$-\frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \int_o^h u \, dy \tag{2}$$

where $\nu = \mu/\rho$ is the kinematic viscosity and $h(x,t)$ is the film thickness.

The boundary and initial conditions are

$$\left.\begin{array}{ll} u(y,o) = 0, & u(o,t) = 0, \\ \dfrac{\partial u}{\partial y} = 0 & \text{at } y = h \end{array}\right\} \tag{3}$$

and

$$h(o,t) = 0 \tag{4}$$